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# DIFFERENTIAL EQUATIONS ONLINE: AN INTERACTIVE VISUAL APPROACH

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**Abstract**
*At the* University of South Australia, *current educational principles have been applied coupled with new technologies in the development of online modules for the course* Differential Equations*. This graphical, interactive courseware integrates constructivist guidelines by using a student centred approach with innovative, inbuilt testing in the expectation that the resulting online learning environment will be highly effective. The modules have being trialled on students, with observations and feedback forming an indication that features of these learning tools such as online quizzes, interactive graphs, comprehensive navigation and real-world examples have contributed to their positive impact upon student learning and satisfaction.*

**Keywords**
*mathematics, online, differential equations, constructivism*

## Introduction

The course *Differential Equations*, offered at the *University of South Australia* has traditionally been taught in a standard lecture/tutorial mode, emphasising a technical approach that relies on mathematical analysis, algebraic manipulation and textual information. With the emergence of computer aided learning, the opportunity has been taken to explore the benefits of supplementing traditional materials with a set of engaging online modules.

The online modules have been developed by applying constructivist principles to create an interactive, task based learning environment that presents a visual, real world perspective. Graphics and animation are vital features used to increase enjoyment and satisfaction, as well as providing a geometric representation of mathematical content to improve conceptual understanding. Engagement is enhanced by high levels of interactivity and innovative forms of assessment that require the application and exercise of new knowledge.

The success of the online modules has been measured through observations during practical sessions and a questionnaire given to students during the final week of semester. Questionnaire responses indicate a perception that the modules fulfil their underlying constructivist principles, with high levels of satisfaction regarding the online components of the course and positive attitudes when compared with written components.

## *Differential Equations*- Course Information

*Differential Equations* is a second year course offered by the *School of Mathematics & Statistics* which aims to serve the purpose of teaching the necessary skills required by any professional mathematician for solving differential equations. Students have similar mathematical ability levels as they have undertaken the prerequisites for the course, two semesters of calculus. Contact time includes three or four hours of lectures per week, and one hour of alternating computer practicals or written tutorials. 20 students were enrolled in 2005, with a 100% completion rate.

## Software and Graphics, presenting a Visual Perspective

Software such as symbolic manipulators, the Internet and graphical calculation packages are beginning to have a dramatic influence on the teaching of mathematics. The focus of course content is moving away from a traditional formula-centred approach to more visual and quantitative analysis (Boyce, 1999). Technology has opened a new way of teaching that allows students to see graphs of solutions with the click of a button. A broad visual overview can be achieved through graphics and animation, movement over time can be seen such as liquid draining from a tank rather than just static, hand drawn steps on a whiteboard. Further, students can play with these solutions, modify parameters, create their own graphs, and be shown realistic models such as the simulation of an oscillating spring.

Currently there is a range of online resources for differential equations, each with a different purpose, which is reflected in the wide variation of content and quality. Some websites such as Mathworld (Weisstein) and Eqworld (Polyanin, 2004) present static, non-interactive pages that are mainly content based, with a large amount of textual information. This is sometimes supplemented with problems or modelling examples to test learning such as *Internet Differential Equations* (LoFaro & Cooper, 1996) or static graphs and graphics to aid cognition. A more dynamic approach is taken by other sites such as *Interactive Differential Equations* (West et al., 2000), which contain visual solutions such as graphs of solution curves. Varying degrees of interactivity include user ability to modify parameters, functions, rotational views of three dimensional models or behaviour over time. These sites would be far more assistive learning tools if they had more scaffolding to guide students through the concepts and materials, accompanied by inbuilt testing.

## Online Modules

A set of online modules have been created that are comprehensively tailored to the course *Differential Equations*, offered at the *University of South Australia*. Developed in Macromedia Flash MX, they have extremely lightweight file sizes despite their highly graphical content. Each module is broken into walkthroughs that introduce new materials and concepts in an interactive format followed by assessment tasks that apply and test new knowledge. Some important goals for the online modules that have been achieved include-

- An interface that is visually appealing,
- Minimisation of text to reduce extensive reading,
- Autonomy of learning, given the student has an appropriate level of prior knowledge.

The modules adhere to many widely supported constructivist principles and guidelines (Skemp, 1986) in order to create an effective online learning environment, which stem from the following key definitions of constructivism-

- Each person forms their own representation of knowledge,
- Learning occurs when a learner uncovers an inconsistency between their current knowledge representation and a new experience,
- Learning occurs within a social context, and interaction between learners and their peers is a necessary part of the learning process.

For a more extended discussion of constructivism as applied to these learning materials, please see (Jacobs 2005).

The modules aim to accommodate a wide range of perceptual styles of learning (Husch, 2001). Information is provided both visually and verbally, in an environment that suits active participation. Students are given the opportunity to work individually or collaboratively. Tasks are well-defined, but often draw together complex relationships.

### Walkthroughs

Each walkthrough introduces new concepts and definitions in a simple, step-by-step format. Information is governed by a clearly labelled navigational structure, with a vertical linear sequence to guide users through the pages as well as providing the flexibility to skip forwards and backwards. These features follow the constructivist guidelines of student centred learning (Walczyk & Ramsey, 2003), by guiding students through materials while still allowing them to control their own navigational sequences, and providing new concepts which have a basis for assimilation into existing knowledge structures (Jonassen & Duffy, 1993).

The use of graphics such as a swinging pendulum and interactive graphs that allow users to modify parameters adheres strongly to three constructivist principles. Students are encouraged to actively explore and interact with these graphs and animations via sliders, which allow them to see the effect of changing different parameters. A dynamic visual perspective is presented as well as values of parameters and analytic solutions (Skemp, 1986). Furthermore materials are situated in a real world context, a vital aid in showing the relevance and application of mathematical knowledge (Cooper & LoFaro, 1999).

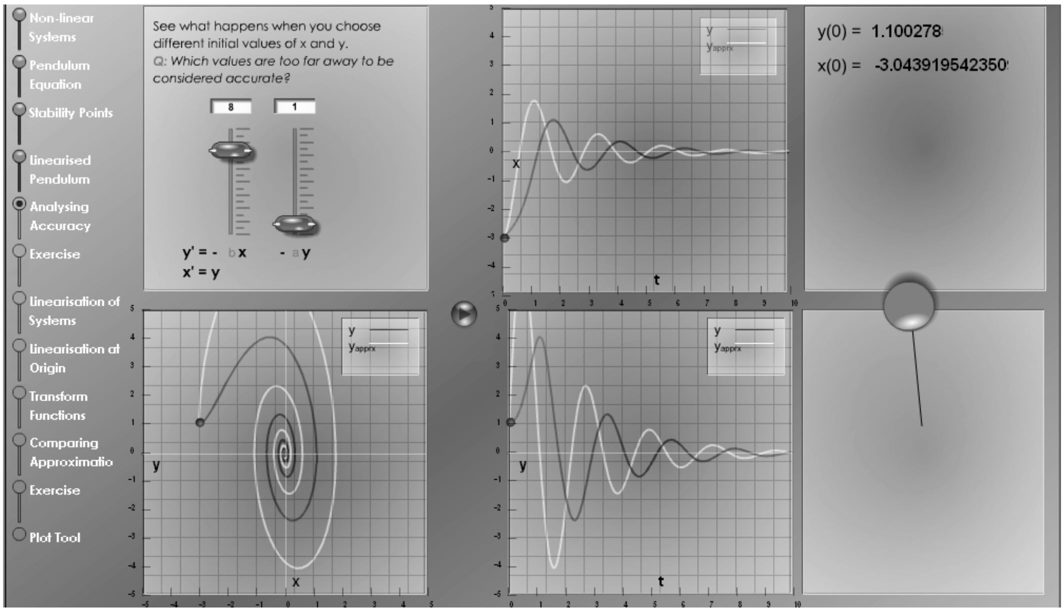


*Figure 1. Nonlinear Systems Walkthrough*

### Online Assessment

Formative and summative assessment tasks have been developed that directly relate to the walkthroughs, providing an experiential basis for testing and applying new knowledge. Automatic corrections and feedback gives students the ability to immediately check their own progress. Randomised multiple choice quizzes and randomised worksheets are the two innovative forms of online assessment that have been used. These assessment tools also aim to address plagiarism issues, with different randomised question “types” students are deterred from directly copying or sharing exact answers, instead they are able to freely discuss problem solving techniques.

### Online Multiple Choice Quizzes

The online multiple choice quizzes not only feature randomised parameters, but also inbuilt, adjustable graphs. This allows questioning on conceptual understanding of graphs and relationships between parameters. The quizzes work on the basic principle that practise makes perfect, by retesting incorrectly answered questions using a new randomly selected set of numbers.

### Online Worksheets

The randomised online worksheets consist of multiple answer questions that are sequentially structured to form the entire working of a lengthy problem. The walkthroughs demonstrate the four steps of scaffolding. Initially answers are presented to students, demonstrating how to arrive at a solution. In the second phase, a similar problem is posed, and students are required to determine the answer with feedback given immediately. The third phase involves the student answering an entire lengthy problem, with instant feedback. The final phase has automatic marking displayed only after the exercise has been completed.

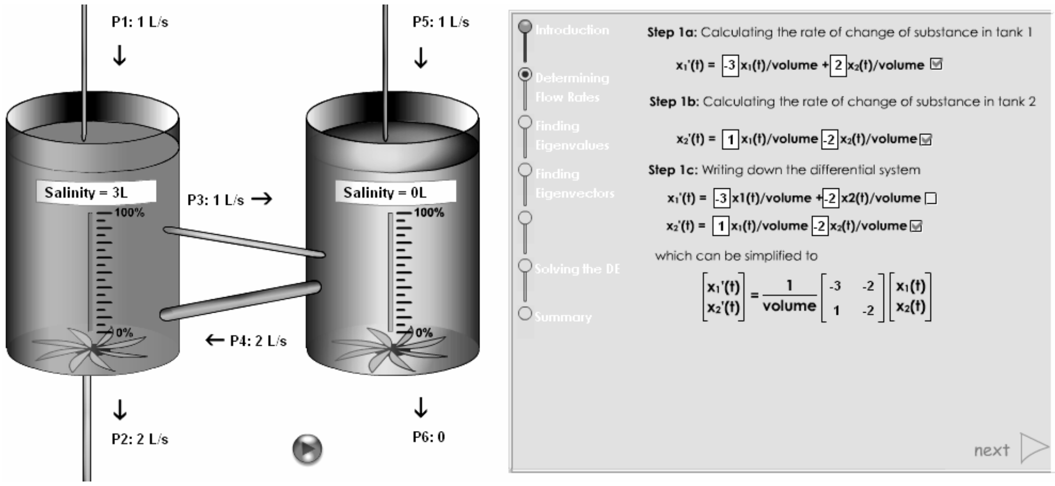


*Figure 2. Water Tank Worksheet*

### ODEToolkit, a Tool for Modelling

The first module introduces ODEToolkit, a software package recently developed by Borrelli and Coleman that is specifically designed for easily setting up and automatically solving differential equations. The software enables students to visualise results, and experiment with the effects of changing system parameters or initial conditions. Its introduction gives students a tool for independently verifying their own analysis and calculations when doing written tutorials and assignments.

## Research Methodology

The study has been undertaken using action research (Zuber-Skerritt 1992) as a framework for providing an iterative and incremental foundation for improvement. Methods of evaluating students interacting with modules included participant observations and discussion with students by researcher/tutor during practicals and an examination of student responses to questionnaires and their academic performance. Analysis of these will form a basis for the refinement, enhancement and additional functionality of the modules.

### Results

Results were collected from questionnaires handed out to all students during the final week, (see Appendix 1). During practical sessions, observations were made by the researcher regarding student performance on the quizzes, opinions voiced about various aspects of the modules, difficulties experienced working with the modules, questions asked of the researcher/tutor and of other students and the levels of interactivity displayed as they worked through the modules.

### *Questionnaire Results*
The questionnaire was divided into four major headings that surveyed opinions on the use of the online modules, a comparison of online assessment and written assignments, a comparison of the online modules and written tutorials and finally a general basis for mathematical preferences. It aimed to address the underlying constructivist principles of the modules. Of the 20 students in the class, 18 returned the questionnaire, one which was void.

### *Using the Online Modules*
Almost all students answered positively in the questionnaire to the use of the interactive features of the online modules. Students generally gave strong positive responses towards the visual appeal, and almost all students indicated that they could easily navigate through topics within modules. Results showed that analysis of graphs and use of sliders generally helped to provide insight into concepts and behaviour of solutions. Responses suggested that real world examples did indeed aid most students in realising the relevance of different topics.

Observations in practicals showed the students interacting with graphs and animations as well as modifying the sliders, particularly when trying to answer quiz questions. Students verbally expressed their appreciation that they could better understand concepts through this approach which had been difficult to comprehend when presented algebraically in lectures. Navigation caused no visible problems, and students appeared to flip easily between topics.

Questionnaire responses showed that students generally felt neutral about the level of detail within explanations. Some practicals were attempted before the material was covered in lectures. This sometimes led to confusion, as the modules do not give detailed mathematical explanations. However students were still able to understand many parts of these practicals. Ideally the walkthroughs would be presented in lectures as a general overview of concepts, in conjunction with more detailed mathematical analysis (in the same way that a power point slideshow is used in a presentation).

Only two students indicated in the questionnaire that they felt there was too much text to read. It was observed that some students preferred to print all information and refer to hard copies. Printable versions of all online materials would be useful to help these students.

### *Comparing Online Assessments with Written Assignments*
Questionnaire responses showed that students generally felt that the online quizzes helped them learn, and they liked the quizzes. Only one student felt that they had guessed the answers to the online quizzes, whereas most responded that they did not guess the answers. Responses indicated that knowledge learnt in walkthroughs was applied in assessments. Many students expressed a preference for the online assessments over written assessments, with only two students disagreeing. Furthermore, students largely felt less stressed by knowing their grades in the assessments with automatic marking.

Initial observations have shown that students did complete the quiz slowly and carefully, using the walkthrough to help them understand different solutions as well as a pen and paper to calculate numerical solutions. Perhaps they may intuitively guess based on prior knowledge, but these observations show that an incorrect guess and later re-testing of the same question type lead to students re-thinking and further analysing the walkthrough before answering again. Students expressed surprise at obtaining full marks despite multiple attempts on some questions, and it was suggested to some that they could repeat the quiz to get a better understanding, which they did.

Observations during practicals showed that the students had only minor difficulties using the online worksheets. Problems mainly arose from the complexity of correct solutions and were due to undetected and easily correctable minor programming errors.

Many students verbally expressed their opinions that the first written assignment was long and difficult. Despite some extremely capable students in the class, none managed to achieve full marks. This contrasted with the automatically marked online assessments that were generally completed during the one-hour practical session, where all students managed to achieve near or full marks. The difference in performance with these two types of assessment is a reflection on their nature, as the online assessments give students an opportunity to continue practising until a correct answer is given as opposed to written assignments which give feedback only after marks had already been assigned.

### *Comparing Online Modules with Written Tutorials*

The autonomy of modules was demonstrated in the questionnaire by students' response that they did not need much assistance from an instructor to do the modules, in contrast to a general indication that they could not do tutorials without explanation. This was also observed during the practical sessions, where students asked few or no questions and were able to complete most or all parts of practicals without assistance. Often the need for assistance was due to materials that had not been covered in lectures or minor errors within the programs that were not detected during trial phases.

ODEToolkit was a software package introduced in the first practical to give students a tool for analysing solutions of differential equations. Approximately a third of students responded in the questionnaire that they used this software to help them understand written tutorials or assignments, which was not explicitly required. During the practical most students had relatively few problems using ODEToolkit and some expressed an opinion that they realised why it was useful. The main problems using the software related to the numerical difficulties with solving differential equations, such as singularities.

In the questionnaire, most students indicated that they enjoyed using the online modules more than doing the tutorial exercises. Students generally indicated a preference for a mix of online modules and written tutorials, and most disagreed with having only tutorials or only online modules. This is understandable as some components of the course are more suited to explanation using a whiteboard in a tutorial whereas others can be more easily explained through a computer aided visual, graphical approach.

### *Mathematical Preferences*

Questionnaire responses indicated a general enjoyment for mathematics however enjoyment for the course was substantially more neutral, despite general positive attitudes towards the online modules. Many factors impact upon student satisfaction within a course, such as difficulty, workload, content, organisation and the modes and quality of delivery. It is suggested that this neutral response is caused by one or more of these factors rather than the introduction of the online components, however inclusion of extra questions will be considered for further investigation.

Student questionnaire responses showed mixed views of doing mathematics on the computer. This may be due to the student cohort comprising of a large number of students who did not have a strong IT background.

Student opinions in the questionnaire regarding the ease of mathematics courses offered at the university were quite varied, ranging from some agreement, to strong disagreement. More than half of the students agreed that they found the *Differential Equations* course difficult, perhaps a key factor that impacted on their level of enjoyment.

### *Interactivity between Students*

The randomized nature of assessments encouraged students to work together without being able to directly copy. It was interesting to note that during practicals students would describe their own qualitative understanding, or techniques for solving a problem rather than directly telling each other the answers.

### *Improvements*

Despite the overall positive response from students towards the online modules, there are still improvements to be made. A few students chose to print online materials and it would be beneficial to provide print friendly versions of all materials. Small errors in programs as well as typing mistakes were uncovered through student usage which need to and are able to be updated. Further functionality could include server interactivity such as the MacQTex quizzes developed by Macquarie University (Griffin & Moore 2003), to capture valuable information such as log on times, completion times and the number of attempts if an assessment is attempted on more than one occasion. General course management is also an issue to be resolved, as better synchronization between practicals and lectures will give students the required prior knowledge for practicals. The use of the online modules during lectures would also be assistive in explaining the materials covered.

## Conclusion

This paper has discussed the effectiveness of supplementing the course *Differential Equations* with online modules. These engaging, interactive modules have been developed using constructivist principles to satisfy the educational requirements of providing an effective learning environment. Enjoyment, understanding of concepts and ease of use were some of the positive feedback indicators showing the success of these modules from the students' perspective. Randomized assessments have provided some valuable barriers to plagiarism by preventing the ability to directly copy and opening up more productive formats for student collaboration. Whilst results indicate that a positive step towards online learning has been achieved, responses show that students do not want traditional tutorial modes of teaching to be abolished, instead indicating their preference for a combination of both practicals and tutorials.

## References


Boyce, W.E. (1999). *Differential Equations in the Information Age.* The Mathematics Association of America.

Cooper, K.D. & LoFaro, T. (1999). *Differential Equations on the Internet.* The Mathematics Association of America.

Griffin, F. & Moore, R. (2003). *MacQTeX Randomised Quiz System*. Paper presented at the Apple University Consortium Conference, Adelaide.

Husch, D.S. (2001). An investigation of the relationships between learning styles, personality temperaments, mathematics self-efficacy, and post-secondary calculus achievement. *Education, Mathematics* (0280). The University of Tennessee.

Jacobs, K. (2005). *An Investigation of Interactive Online Visual Tools for the Learning of Mathematics.* Paper presented at the Kingfisher Delta '05, The Fifth Southern Hemisphere Symposium on Undergraduate Mathematics and Statistics Teaching and Learning, Fraser Island, Queensland, Australia.

Jonassen, D.H. & Duffy, T.M. (1993). *Constructivism and the technology of instruction : a conversation.* Hillsdale, N.J.: Lawrence Erlbaum Associates Publishers.

LoFaro, T. & Cooper, K. (1996). *IDEA- Internet Differential Equations Activities.* Brooks/Cole Publishing. [Online] Available:<http://www.sci.wsu.edu/idea/>. [20th June 2005].

Polyanin, A.D. (2004). *EqWorld- The World of Mathematical Equations.* Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow. [Online] Available: <http://eqworld.ipmnet.ru/eqworld/>.

Skemp, R.R. (1986). *The psychology of learning mathematics, 2nd edn.* Harmondsworth, England: Penguin.

Walczyk, J. & Ramsey, L. (2003). Use of Learner-Centered Instruction in College Science and Mathematics Classrooms. *Journal of Research in Science Education, vol. 40, no. 6.* 566-584.

Weisstein, E.W. *Mathworld.* Wolfram Web Resources. [Online] Available: <http://mathworld.wolfram.com/>.

West, B., Strogatz, S., McDill, J.M. & Cantwell, J. (2000). *Interactive Differential Equations.* Addison Wesley Longman. [Online] Available: <http://www.aw-bc.com/ide/>.

Zuber-Skerritt, O. (1992). *Action research in higher education : examples and reflections.* London: Kogan Page.

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## Appendix 1- Questionnaire Response Frequencies

| **Using the Online Modules** | **SA** | **A** | **N** | **D** | **SD** |
|---|---|---|---|---|---|
| 1. I liked the interactive features of the modules (e.g. animated spring and pendulum, adjustable graphs etc). | 7 | 9 | 1 | 0 | 0 |
| 2. I found the modules visually appealing. | 7 | 8 | 2 | 0 | 0 |
| 3. I could easily navigate through the topics within each module. | 6 | 8 | 2 | 1 | 0 |
| 4. Analysing graphs helped me to understand the concepts. | 5 | 9 | 3 | 0 | 0 |
| 5. Using the sliders helped me to see the effect of modifying a parameter on the solution. | 3 | 12 | 2 | 0 | 0 |
| 6. Real world examples (e.g. animated spring, pendulum etc) helped me to realise the relevance of different topics. | 5 | 8 | 2 | 2 | 0 |
| 7. Explanations were not detailed enough. | 0 | 4 | 10 | 3 | 0 |
| 8. There was too much text to read. | 0 | 2 | 4 | 10 | 1 |

| **Comparing Online Assessments with Written Assignments** *(including the ODEToolkit assignment, Mass on a Spring quiz, Water Tank assignment, Eigenvectors & Eigenvalues quiz, and Fourier series assignment)* | **SA** | **A** | **N** | **D** | **SD** |
|---|---|---|---|---|---|
| 9. I felt that the online quizzes helped me learn. | 1 | 14 | 2 | 0 | 0 |
| 10. I guessed answers to the online quizzes. | 0 | 1 | 4 | 11 | 1 |
| 11. I liked doing the online quizzes as a form of assessment. | 3 | 11 | 3 | 0 | 0 |
| 12. I preferred the online assessments rather than the written assignments. | 4 | 7 | 4 | 2 | 0 |
| 13. I felt less stressed by knowing my grades for the online assessments with automatic marking. | 8 | 5 | 3 | 1 | 0 |
| 14. The assessments helped me to apply the knowledge learnt in the walkthroughs. | 2 | 9 | 6 | 0 | 0 |

| **Comparing Online Modules with Written Tutorials** | **SA** | **A** | **N** | **D** | **SD** |
|---|---|---|---|---|---|
| 15. I could use the online modules without much assistance from an instructor. | 2 | 11 | 4 | 0 | 0 |
| 16. I could do the tutorials without needing any explanation from an instructor. | 0 | 4 | 5 | 6 | 1 |
| 17. I enjoyed using the online modules more than doing tutorial exercises. | 3 | 8 | 6 | 0 | 0 |
| 18. Using the online modules in addition to the tutorials helped me to understand the concepts better. | 3 | 9 | 5 | 0 | 0 |
| 19. I used ODEToolkit to help me understand solutions of equations for written tutorials or assignments. | 1 | 6 | 3 | 7 | 0 |
| 20. I would prefer to have- online modules (without written tutorials) | 0 | 3 | 7 | 4 | 1 |
| written tutorials (without online modules) | 0 | 2 | 6 | 5 | 2 |
| both online modules and written tutorials | 7 | 6 | 4 | 0 | 0 |

| **Mathematical Preferences** | **SA** | **A** | **N** | **D** | **SD** |
|---|---|---|---|---|---|
| 21. I enjoy mathematics. | 6 | 9 | 2 | 0 | 0 |
| 22. I enjoyed taking this course. | 1 | 2 | 12 | 2 | 0 |
| 23. I like doing maths on the computer. | 1 | 6 | 6 | 3 | 1 |
| 24. I generally find the mathematics courses I have done at university easy. | 0 | 4 | 7 | 4 | 2 |
| 25. I found this course easy. | 0 | 2 | 7 | 6 | 2 |

*Table 1. Frequency of Questionnaire Responses*