International Journal of Mathematical Education in Science and Technology, Vol. 36, No. 7, 2005, 761–768

Taylor & Francis
Taylor & Francis Group

Investigation of interactive online visual tools for the learning of mathematics

K. L. JACOBS*

School of Mathematics & Statistics, University of South Australia, Mawson Lakes, SA 5095, Australia

(*Received 2 May 2005*)

For many years, educators have been discussing benefits of educational practices such as the use of real-world examples, visualisation, interactivity, constructivism, self-paced learning and self-paced testing. Macromedia Flash MX has been used to develop online modules for the course Differential Equations offered at the University of South Australia that utilize these practices. This paper demonstrates the following innovative features of the modules that provide an effective online learning environment:

- graphs which allow the user to interactively vary parameters are accompanied by 'walkthroughs' that break mathematical content into manageable chunks;
- a clearly labelled navigational structure that implies a linear order for each module while allowing users to jump forwards or backwards;
- animations such as an oscillating spring which depict real-world examples.

Online multiple-choice quizzes with randomized numeric values is a unique form of assessment that will be discussed. The integration of a wide range of effective learning principles into online modules will be the focus of this paper.

1. Introduction

Mathematics is one of the most difficult fields to learn as it requires both intuitive and analytic intelligence [1]. As the number of people studying mathematics declines each year, it is becoming increasingly important to encourage students, and present materials in a way that maximizes learning, enjoyment and satisfaction. Traditional lectures place a strong emphasis on algebra, analysis and textual information. Research shows that a more visual, constructivist approach which demonstrates the geometric representation of concepts as well as their technical significance enables students to gain more insight into the underlying meaning [2].

The introduction of computers, internet and online learning gives educators the opportunity to create courseware that is visually captivating. A geometric overview of mathematics can be presented that is superior to any hand drawn graph, and engagement can be enhanced with the additional features of interactivity and animation. Some universities are realizing the impact of using computers and technology

*Email: Kuva.Jacobs@unisa.edu.au

International Journal of Mathematical Education in Science and Technology
ISSN 0020–739X print/ISSN 1464–5211 online
http://www.tandf.co.uk/journals
DOI: 10.1080/00207390500271149

to teach differential equations. However, few currently incorporate emerging software into a complete online course.

The aim of this project has been the development of an online Differential Equations course that is provided by the University of South Australia. Modern educational principles which are widely believed to support learning have been applied, coupled with innovative new technologies. A suite of online modules have been created to enhance standard lectures and tutorials, by providing courseware that presents materials in an appealing, interactive format, allowing students to explore new concepts from a visual perspective.

2. Constructivist framework for learning

Modern educational practices have moved away from traditional teacher-centred environments, favouring the adoption of a constructivist, student-centred approach [1]. The constructivist perspective of learning can be defined by three broad principles

- Each person forms their own representation of knowledge.
- Learning occurs when a learner uncovers an inconsistency between their current knowledge representation and a new experience.
- Learning occurs within a social context, and interaction between learners and their peers is a necessary part of the learning process [3].

A number of the following constructivist principles and guidelines have been applied to this project in the design of effective instructional tools.

2.1. *Zone of proximal development*

Students should be provided with new concepts that have a basis for assimilation into existing knowledge structures, so that materials are challenging but possible to understand with appropriate guidance [4]. This is known as their Zone of Proximal Development [5], and can be achieved by introducing new ideas step-by-step, starting with a simple case and building up to complex examples.

2.2. *Concrete instruction in a world of concrete*

New concepts should be situated in a real-world context, allowing students not only to grasp concepts (what the problem and solution is) and the processes involved (how they solve it) but also the application of these concepts (why they need to solve these problems). Materials should be objective driven, with 'hands on' experiential activities to stimulate motivation and enthusiasm. Realistic models such as the simulation of a pendulum swinging give abstract concepts concrete meaning.

2.3. *Cross-connections*

The use of multiple perspectives such as both visual and verbal explanations reinforces learning by providing multiple connections. Imagery helps students understand structure and relationships [3]. A broad visual overview can be achieved through graphics and animation, movement over time can be seen rather than just static, hand drawn graphs on a blackboard. Visual insight can be obtained through graphing that gives even non-mathematicians the capability to analyse solutions before a more technical phase is reached.

Cognitive theories further promote the use of graphics and visual tools, particularly Paivio's dual coding theory [6]. This theory describes two separate cognitive subsystems, one subsystem processing imagery and the other able to process verbal or textual information. It is suggested that information is more readily processed and retained through presentation of both images and text.

2.4. *Assessment*

Learning should be experiential, with testing and application of knowledge within a relevant problem domain. Therefore modules are supplemented with suitable exercises that test understanding as both formative and summative assessment tasks. These exercises follow the outlined principles of testing multiple viewpoints within a real world context and automatic feedback provides students with a challenging basis for measuring their own understanding.

2.5. *Students take control*

Constructivism emphasizes the idea of student-centred learning, with students taking an active approach to their own learning rather than passively receiving information. They should be encouraged to explore, investigate and question any new concept presented to them. Consequently, an educator should act as a guide, scaffolding and facilitating learning. Online instruction can adapt these principles by providing a strong level of interactivity which allows students to take control of their own learning. The ability to choose navigational sequences, interact with solutions, modify parameters, and create graphs are examples of an approach that encourages student-centred learning. Furthermore, the development of learning tools that are perceived by students as enjoyable to use, hence enhancing intrinsic motivation will stimulate interest and create a more active learning environment [7].

3. Development

The number of online higher level mathematics resources is growing rapidly. Web-based references, such as Mathworld [8] and the latest Eqworld [9] are an excellent place to quickly find a formula but are generally not aimed to be used as instructional tools. Other websites such as Interactive Differential Equations [10] contain useful Applet tools, allowing users to explore graphical solutions with minimal guidance. These Applets would be more assistive learning aids with a stronger level of scaffolding, clear explanations of concepts and activities to apply and test knowledge.

Effective examples of an autonomous approach to learning in different areas are the BBC languages website [11], targeted to help people learn basic speaking skills within a language and BrainPOP [12], an educational children's website. Both websites feature an online guide with step-by-step information and interactive testing with automatic checking. Some key features of these websites include the minimization of text to prevent the learner from becoming distracted or overwhelmed and the use of quality designs with high levels of colour, to help maintain attention by enhancing visual appeal.

3.1. *Online modules*

Differential equations textbooks are beginning to emphasize more and more modelling and graphical techniques in conventional courses, which provide an active approach to learning. The project has taken this approach one step further, producing a set of independent online modules that introduce students to mathematical modelling using differential equations in a package comprehensively tailored to their course. Modules are based on concepts relevant to students' areas of study rather than rote learning new solution techniques, and include extensive animation and other visual aids to maximise comprehension. The inclusion of interactive elements enables students to investigate and explore beyond the capability of current teaching aids. The modules are introduced as a supplement to traditional lectures, tutorials and a text book. Developed in Macromedia Flash MX, they are extremely lightweight, the largest having a file size of 120 KB.

The provision of a highly graphical view of mathematics enables students to explore concepts and link these with previous learnt theory. Each module aims to be self-explanatory and while lacking in extensive textual detail, it should still be understandable given that the student has the relevant level of assumed knowledge. Text is minimized, emphasizing an overview of concepts rather than providing lengthy descriptions of procedures or theories which can easily be found in a textbooks or lecture notes. This task-based rather than theoretical structure adheres to constructivist learning principles. An engaging, high quality interface is also a necessary aim that has been achieved in order to maintain attention more easily by increasing visual appeal.

3.1.1. Navigational structure. The major navigational structure is governed by a vertical linear sequence of main headings on the left side of each page. This structure is provided as a guide only, giving the student full control to skip forwards or backwards between pages. New concepts, variables or definitions are introduced on each page examining the simplest case and building with each page to more complex cases. All information for one page is visible on the screen, and scroll panes are deliberately not used, to avoid lengthy explanations.

3.1.2. Interactive graphs. Interactive graphs form an important element of the online modules. Students are given control of sliders to change the value of certain parameters. They can immediately discover the effect of this modification as the solution to the new differential equation is plotted, encouraging them to explore and investigate. These sliders enable the formation of very important conceptual understanding of boundary conditions, such as the change of one variable to form a solution with real, repeated or imaginary roots that is not easily achieved with a static graph. An analytic solution as well as a graph is provided for solvable differential equations, which enables students to understand how this solution affects the graphs, such as the presence of trigonometric, or exponential solutions.

3.1.3. Animations. Animations such as a shiny mass on a spring, or water tanks that fill and empty with yellow liquid are not only included as a stimulating way to catch students' attention. They provide students with a real world simulation that is an important conceptual bridge between reality and the graphs and equations of a standard mathematics course. Students can see a pendulum with angle exactly π and velocity 0 is balanced upside down and will not move unless given a slight push.

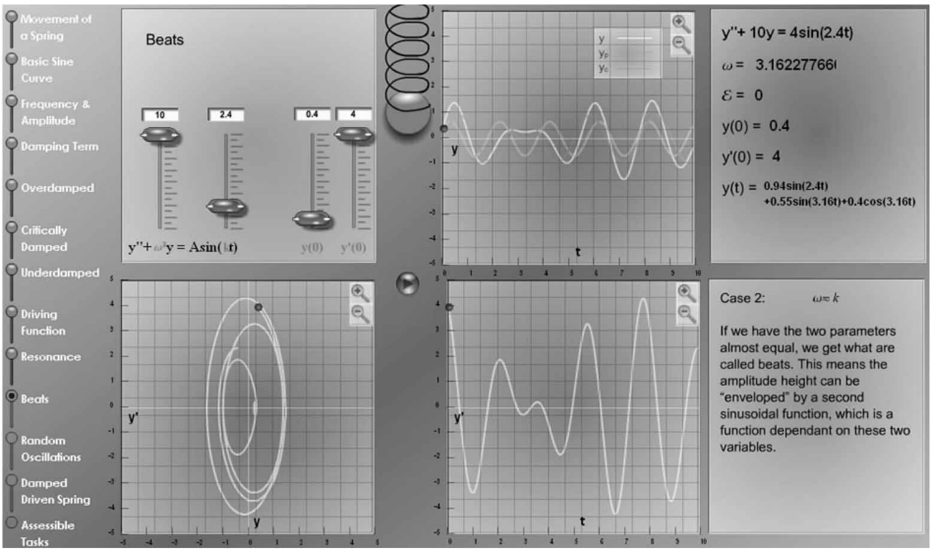

Figure 1. Online 'mass on a spring' worksheet.

3.2. *Case study – Module 2 – Mass on a spring*

The 'mass on a spring module' introduces the differential equation describing the movement of a mass on a spring. The simplest case is initially examined, with observations of the effects of the initial position and velocity. Later damping and finally a driving function are introduced, with a careful examination of the relationships between variables and their effect on the solution.

These solutions are plotted in three adjacent graphs, showing the position over time $y(t)$, velocity over time $y'(t)$ and orbital portrait $y'(y)$. Animated dots depict the graphs as functions of time as well as in Cartesian coordinates. Text is minimized, with further discussion separated from the main panel, allowing the rapid acquisition of key terms and ideas. Parameter values and the differential solution are displayed in the top right panel. An animated spring is included, as a real-world example of what the mathematical graphs and equations are describing.

3.3. *Online assessment*

An important aspect of learning is the exercise of new knowledge and skills within a relevant problem domain. Online assessments test student understanding, by transferring and applying knowledge learnt in walkthroughs to suitable problems. Relevant feedback is provided either immediately or upon completion of a stage of the assessment, to enable the student to recognize and correct errors as well as building confidence when they give a valid response. The two types of assessment developed are randomized multiple choice quizzes and randomized worksheets. They are both designed to be completed in conjunction with the online walkthroughs and all questions directly relate to concepts and materials covered in the walkthroughs.

Randomized parameters allow users to repeat multiple question types. Prompting students to repeat question types until they are able to answer correctly has two positive effects. Students are encouraged to answer correctly the first time so they do not need to do extra work and also to keep trying if they do not understand immediately. Practise and repetition encourages students to find the right solution and reflect on incorrect choices.

Randomization has further advantages in terms of avoiding plagiarism. Students can compare and discuss the different question 'types' for each question without giving an exact answer. Each assessment requires the student to initially enter their student identification which is displayed upon completion. Additional measures taken to deter plagiarism include the use of gradient backgrounds that prevent students from easily copying and manipulating digital images.

3.3.1. Online multiple-choice quizzes. Randomized quizzes are a form of assessment that is becoming more widely practised. The most common form of randomisation involves creating a large set of questions and selecting a random subset each time the quiz is performed by a student.

Some mathematics quiz systems have developed sophisticated techniques for creating questions with randomized parameters such as MacQT$_E$X which was recently created by Macquarie University [13]. This package uses programs and server interactivity to generate PDF quizzes, enabling automatic marking and retrieval of student scores and information via a server. Questions are already written, and the lecturer can only choose from the provided set. Griffin and Moore highlight their MacQT$_E$X quiz system as having several advantages over other similar quiz systems including the use of PDF output as opposed to HTML, the requirement of server interaction only upon initialization and completion of a quiz as opposed to continuous communication with a server, and a stand alone system that does not require algebra packages or databases. The disadvantages of this quiz system are large initial downloads, lack of connections between questions and an inability to include dynamic graphics.

The set of randomized online multiple choice quizzes for this course are tailored to complement the online walkthroughs, featuring small file sizes (32 KB) and random generation of parameters on the fly, using embedded programming. Graphics are included, allowing a focus on questions which test conceptual understanding of graphs and relationships between parameters as opposed to lengthy algebra questions that are easier to solve using pen and paper.

The quizzes continue testing students until the correct answer is eventually achieved for all question types. The quiz steps through each question type, prompting the correct selection of three possible answers. Upon completion of all question types, a student will return to those previously answered incorrectly, which are re-tested with a different set of numbers. This process continues until finishing, when all questions have been answered correctly. A summary screen presents the number of attempts for each question to be printed and handed up. Full marks are awarded upon completion.

One weakness of this form of assessment is the possibility for students to simply guess solutions. Indeed, a poor student with little concern for understanding their subject may simply guess, and this random selection will quickly result in completion (by continuously pressing the 'next' button the quiz can easily be completed within five minutes). A student who actually wants to understand their course will perceive any assessment to have educational value. Furthermore if they are conscientious, they will want to answer any question correctly and consider it carefully before answering. Initial observations have shown that students did complete the quiz slowly and carefully, using the walkthrough to help them understand

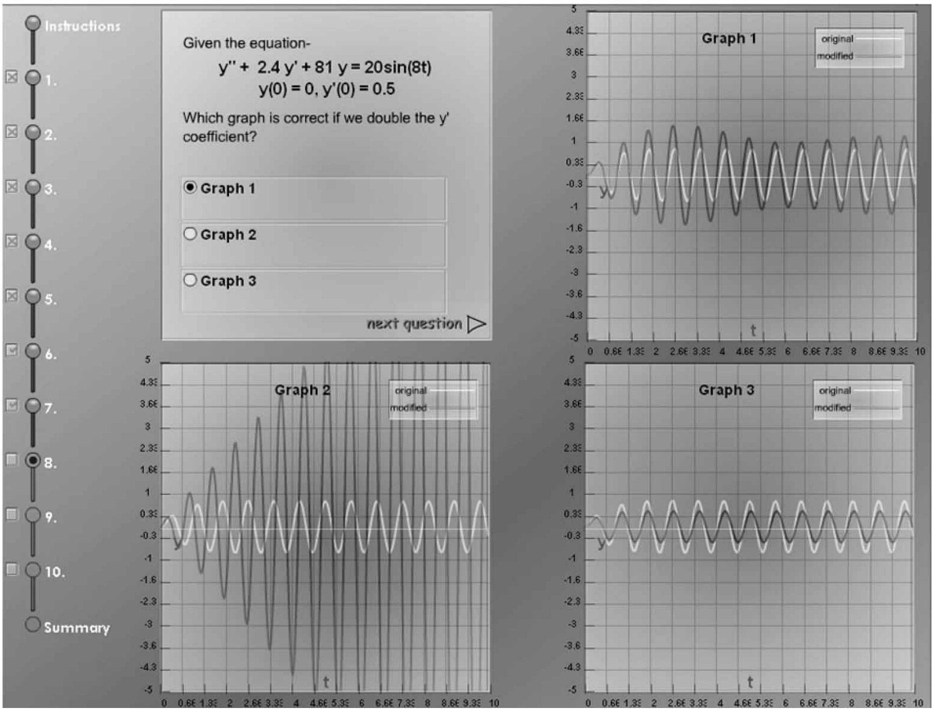

Figure 2. Online 'mass on a spring' quiz.

different solutions. Perhaps they may intuitively guess based on prior knowledge, but these observations show that an incorrect guess and later re-testing of the same question type lead to students re-thinking and further analysing the walkthrough before answering again.

3.3.2. Online worksheets. The online worksheets include multiple answer questions that are sequentially structured to form the entire working of a lengthy problem. The two types of automatic marking include ticks that appear as the student inputs a section of a page correctly and a marking scheme that displays marks for each section only upon completion of all sections. Both schemes allow students to revise their answers in order to achieve full marks. In contrast to the quizzes, the open style questions of the worksheets are not easily guessed. This scaffolded scheme gives a clear framework for solving mathematical problems, focusing on both modelling concepts and the associated analytic problem solving procedures.

4. Conclusions

This paper has discussed the application of constructivist learning principles to the design of online modules for the teaching of differential equations. A visual, interactive format has been used to develop modules that are supplemented by innovative randomized assessments with automatic marking. Observations of students interacting with modules and analysis of feedback are measures currently being implemented that will form a basis for evaluating the effectiveness of these modules.

References

[1] Douville, P. and Pugalee, D.K., 2003, Investigating the relationship between mental imaging and mathematical problem solving, in *Mathematics Education into the 21st Century Project* (Brno, Czech Republic).
[2] Borrelli, R.L. and Coleman, C.S., 1999, Modeling and visualization in the introductory ODE course, in M.J. Kallaher (Ed.) *Revolutions in Differential Equations, Exploring ODEs with Modern Technology* (Reston, VA: The Mathematics Association of America).
[3] Skemp, R.R., 1986, *The Psychology of Learning Mathematics*, 2nd edn (Harmondsworth, UK: Penguin).
[4] Jonassen, D.H. and Duffy, T.M., 1993, *Constructivism and the Technology of Instruction: a Conversation* (Hillsdale, NJ: Lawrence Erlbaum Associates).
[5] Vygotsky, L.S., 1978, In M. Cole (Ed.) *Mind in Society: the Development of Higher Psychological Processes* (Cambridge, MA: Harvard University Press).
[6] Paivio, A., 1971, *Imagery and Verbal Processes* (New York: Holt Rinehart and Winston).
[7] Craig, R.C., Mehrens, W.H. and Clarizio, H.F., 1975, *Contemporary Educational Psychology: Concepts, Issues, Applications* (new York: John Wiley & Sons).
[8] Weisstein, E.W., 2001, *Mathworld*, Wolfram Web Resources, http://mathworld.wolfram.com/.
[9] Polyanin, A.D., 2004, *EqWorld–The World of Mathematical Equations* (Moscow: Institute for Problems in Mechanics, Russian Academy of Sciences). http://eqworld.ipmnet.ru/eqworld/.
[10] West, B. *et al*., 2000, *Interactive Differential Equations* (Addison Wesley Longman), http://www.aw-bc.com/ide/.
[11] *BBC Languages*, 2002 (London: BBC). http://www.bbc.co.uk/languages/.
[12] *BrainPOP*, 2004, BrainPOP.com LLC. http://www.brainpop.com/.
[13] Griffin, F. and Moore, R., 2003, MacQTeX randomised quiz system. In *Apple University Consortium Conference,* Adelaide.